

# Quantum Field Theory Vector Spaces

## Operators

Physical quantities like energy, electric charge, momentum, spin, weak isospin charge, QCD charge

$H, Q, \mathbf{P}$ , etc. on states  
 $\Sigma_3, \sigma_i/2$ , etc. on fields

## Fields

Solutions to field equations that create and destroy particles

$\phi, \psi, A^\mu$

## States

Physical particles, one or more (though vacuum state has none)

$|\text{particles}\rangle$   
 like  $|e^+\rangle$  or  $|e^+ e^-\rangle$

Operators  $\longleftarrow$  A vector space and operators  $\longrightarrow$  Vectors

Operators  $\longleftarrow$  Another vector space and operators  $\longrightarrow$  Vectors

Operators  $\longleftarrow$  Yet another vector space and operators  $\longrightarrow$  Vectors

Example  $\Sigma_3 \psi_{\mathbf{p}=0}^{spinup} = \frac{1}{2} \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & 1 & \\ & & & -1 \end{bmatrix} \left[ c_r(\mathbf{p}) \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{-ipx} + d_r^\dagger(\mathbf{p}) \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} e^{ipx} \right] = \frac{1}{2} c_r(\mathbf{p}) \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{-ipx} + d_r^\dagger(\mathbf{p}) \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} e^{ipx} = \frac{1}{2} \psi_{\mathbf{p}=0}^{spinup}.$

Example  $\text{QFT } \Sigma_3 \left| e_{ip}^-(r=1) \right|_{\mathbf{p}_1=0} \rangle = \left( \int_V \psi^\dagger \Sigma_3 \psi d^3x \right) \left| e_{ip}^-(r=1) \right|_{\mathbf{p}_1=0} \rangle$

$$= \sum_{r, \mathbf{p}} \frac{m}{E_{\mathbf{p}}} \left( u_r^\dagger(\mathbf{p}) \Sigma_3 u_r(\mathbf{p}) N_r(\mathbf{p}) + v_r^\dagger(\mathbf{p}) \Sigma_3 v_r(\mathbf{p}) \bar{N}_r(\mathbf{p}) \right) \left| e_{ip}^-(r=1) \right|_{\mathbf{p}_1=0} \rangle = \frac{1}{2} \left| e_{ip}^-(r=1) \right|_{\mathbf{p}_1=0} \rangle$$